

Nanofluid flow and heat transfer in boundary layers at small nanoparticle volume fraction: Zero nanoparticle flux at solid wall

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The continuum formulation is applied to the developing boundary layer problem, which approximates the entrance region of nanofluid flow in micro channels or tubes. The thermophysical properties are expressed as “equations of state” as functions of the local nanofluid volume fraction. Based on experimental utilization of nanofluid prevalently at small volume fraction of nanoparticles, a simple perturbation procedure is used to expand dependent variables in ascending powers of the volume fraction. The zeroth order problems are the Blasius velocity boundary layer and the Pohlhausen thermal boundary layer. These are accompanied by the volume fraction diffusion equation. In detailed applications, the boundary condition of zero-volume flux at a solid wall is specified yields an “insulated wall” solution of constant volume fraction. Two property cases are calculated for comparisons: one is the use of mixture properties for the nanofluid density and heat capacity and the transport properties prevalently used in the literature attributed to Einstein and to Maxwell. Results for Alumina are compared to experiments. The theory under estimates the experimental results. This lead to the second comparison, between “conventional” properties and that obtained from molecular dynamics computations available for gold-water nanofluids. The latter properties considerably increased the heat transfer enhancement relative to that using “conventional” properties and heat transfer enhancement is comparable to the enhanced skin friction rise. To fully appreciate the potential of nanofluids and heat transfer enhancement, further molecular dynamics computations of properties of nanofluids, including transport properties, accompanied by careful laboratory experiments on velocity and temperature profiles are suggested.

Keywords: nanofluids, heat transfer enhancement nanoparticles; boundary layers

1. Introduction

Following an earlier work on the Rayleigh-Stokes solution for nanofluid flow and heat transfer [1], which corresponds to the suddenly accelerated flat plate in its own plane, the present contribution considers the corresponding laminar boundary layer retaining the initial nonlinearity owing to fluid advection. The motivation is for flows with a leading edge where experiments of Wen & Ding [2] and Jung, et al. [3] show significant heat transfer rate intensification compared to the fully developed regions of nanofluid flow in channels and tubes. The nanofluid is here taken as a base liquid, such as water, seeded with dispersed nano sized particles (of order) for purposes of enhancing thermal conductivity and used in micro sized channels or tubes (of the order in width or diameter) for enhancing the surface heat transfer rate in a manner depicted by experiments {2,3}.

The nanofluid is modeled as a liquid containing nanoparticles, small enough so that the nanoparticles are in thermodynamic and momentum equilibrium as if a single fluid. As such, its behavior as a flowing fluid is modeled after a single component “Navier-Stokes fluid”, but that the thermophysical properties are obtained by separate calculations, empirical relations or via measurements pertaining to dilute concentration of nano sized particles embedded in a base liquid. However, in nanofluids, because of Brownian diffusion due to the bombardment of nanoparticles by the basic liquid molecules, the nanofluid description is supplemented by a diffusion equation for nanoparticle concentration (which is conveniently represented by its volume fraction). This is derivable from the continuity equation for the nanoparticle phase in terms of its mass fraction in the nanofluid. Fick’s Law for the diffusion currents relative to a mass averaged flow supplements this. The binary diffusion coefficient is then identified with the Brownian diffusion coefficient. Fick’s law is expressed in terms of the nanoparticle mass fraction and is easily converted to that for the volume fraction for small volume fraction. This thought exercise would obtain the diffusion equation presented by Buongiorno [4], following the formalism presented by Bird, et al. [5] (see also Probstein [6]). Nanofluid description, however, is inescapable from taking on some aspects of mixture of gases [7].

The general description of nanofluid flow has been applied to boundary layer type flows by Pfatsch [8] who numerically integrated the boundary layer equations. On the basis of prevalent experiments that use small volume concentrations of the nanoparticles, on the order of a few percent or less, a perturbation procedure is devised for the Rayleigh-Stokes problem [1]. While this allows the analytical derivation of perturbation results which aided the interpretation of mechanisms involved in the nanofluid flow and heat transfer, the present work will yield mechanisms of advection vs. thermal conduction relevant to the most involved boundary layer problem. The present contribution considers the laminar boundary layer on a flat plate, which would correspond to the Blasius flow [9] in absence of nanoparticles, with the perturbation for small volume concentration applied. The accompanying heat transfer problem corresponds to that of Pohlhausen [9, 10]. It is not possible to obtain analytical solutions in the manor of the Rayleigh-Stokes problem [1] since the basic Blasius problem requires numerical solution.

2. The general formulation

2.1 The surface heat transfer rate

The surface heat transfer rate, in the boundary layer approximation, is first discussed, as this is the important goal we are after,

$$q_0 = - \left(k \frac{\partial T}{\partial y} \right)_0 + (j_p h_p)_0 \quad (2.1)$$

where q_0 is the surface heat transfer rate, k is the nanofluid thermal conductivity, y is normal to the wall coordinate, j_p is the diffusion current of the nanoparticle phase relative to the mass averaged velocity, h_p is the enthalpy of the nanoparticle phase. Subscript 0 denotes evaluated at the wall surface. Equation (2.1) follows from the more general relation of additional heat transfer mechanism owing to diffusional transport of energy, as is in the case of reacting gases [11].

Fick's Law for the diffusion current with respect to the mass averaged velocity in terms of the nanoparticle phase mass fraction X_p is preferred, as this follows the mass conservation relation of the nanoparticle phase which leads to the diffusion equation. The mass fraction is then related to the volume fraction ϕ and the diffusion current in terms of the volume fraction ϕ becomes

$$j_p = - \left(\rho_p D \frac{\partial \phi}{\partial y} \right)_0 + \mathcal{G}(\phi^2) \quad (2.2)$$

where $\mathcal{G}(\phi^2)$ denotes to the order of ϕ^2 , the binary diffusion coefficient D is identified with that of Brownian diffusion [4], ρ_p is the nanoparticle density. The surface heat transfer rate relation [4] expressed in the present simplified boundary layer form is thus

$$q_0 = - \left(k \frac{\partial T}{\partial y} \right)_0 - \left(\rho_p D \frac{\partial \phi}{\partial y} h_p \right)_0 \quad (2.3)$$

It is observed that while enhanced thermal conductivity directly enhances the surface heat transfer rate, it indirectly stretches out the temperature profile so that the temperature gradient is lessened. On the other hand, fluid advection of thermal energy helps steepen the temperature gradient. These mechanisms are pointed out even in the much-simplified Rayleigh-Stokes heat transfer problem [1].

2.2 Continuity and momentum equations

The two-dimensional boundary layer equations for steady flow are written in dimensionless form in terms of asterisked quantities. The global continuity equation is

$$\frac{\partial \rho^* u^*}{\partial x^*} + \frac{\partial \rho^* v^*}{\partial y^*} = 0 \quad (2.4)$$

where the respective dimensionless streamwise and wall normal coordinates, x^*, y^* are normalized by a streamwise length scale L ; u^*, v^* are the respective dimensionless velocity components normalized by the free stream velocity U . The dimensionless nanofluid density ρ^* is normalized by the base fluid density with subscript f , $\rho^* = \rho / \rho_f$. The zero-pressure gradient streamwise momentum equation is

$$\rho^* \left(u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \right) = \frac{1}{\text{Re}} \frac{\partial}{\partial y^*} \left(\mu^* \frac{\partial u^*}{\partial y^*} \right) \quad (2.5)$$

$Re = UL/\nu_f$ is the Reynolds number based on fluid kinematic viscosity $\nu_f = \mu_f/\rho_f$, μ_f is the fluid viscosity coefficient. The nanofluid viscosity coefficient μ is made dimensionless as $\mu^* = \mu/\mu_f$. The wall normal momentum equation reduces to zero pressure gradient across the boundary layer.

2.3 Thermal energy equation

The energy equation is similar to that for a reacting gas in that the heat flux vector would include heat transport of thermal energy by diffusion currents. The generic convective heat transfer equation follows from the first law of thermodynamics in differential form. For the steady two-dimensional boundary layer, written first in dimensional form

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = - \frac{\partial q}{\partial y} \quad (2.6)$$

where $h = \int c dT$ is the static enthalpy of the nanofluid, T is the temperature, C is the nanofluid heat capacity $C = dh/dT$. The energy equation is in the incompressible form for low Mach numbers (i.e., where the Mach number $M \rightarrow 0$) and small temperature loading. For low Mach numbers, the work owing to pressure gradients and the rate of viscous dissipation are negligible as these effects are proportional the square of the Mach number (see Lagerstrom [12]). The small temperature loading allows taking the thermophysical properties to be temperature independent. However, properties such as ρ , C and transport coefficients are nanofluid volume concentration dependent.

The boundary layer form of q in the interior of the nanofluid is

$$q = - \left(k \frac{\partial T}{\partial y} \right) - \left(\rho_p D \frac{\partial \phi}{\partial y} h_p \right) \quad (2.7)$$

Inserting (2.6) into (2.5), the dimensionless energy equation then takes the form

$$\rho^* c^* u^* \frac{\partial \theta}{\partial x^*} + \rho^* c^* v^* \frac{\partial \theta}{\partial y^*} = \frac{1}{Re Pr_f} \frac{\partial}{\partial y^*} \left(k^* \frac{\partial \theta}{\partial y^*} \right) + \frac{\phi_\infty}{Re Sc_f} \frac{\partial}{\partial y^*} \left(\rho_p^* D^* \frac{\partial \Phi}{\partial y^*} C_p^* \theta \right) \quad (2.8)$$

where the dimensionless volume fraction is defined as $\Phi = \phi/\phi_\infty$, ϕ_∞ is the volume fraction in the free stream, $D^* = D/D_{ref}$ where for Brownian diffusion [4] $D = k_B T / 6\pi\mu_f r_d$, k_B is the Boltzmann constant, r_d is the nanoparticle radius (or averaged radius), $D_{ref} = k_B T_{ave} / 6\pi\mu_f r_d$, T_{ave} is an averaged temperature. The dimensionless temperature is $\theta = (T - T_\infty) / (T_0 - T_\infty)$. The base fluid Prandtl and Schmidt numbers are, respectively, $Pr_f = \nu_f / \alpha_f$ and $Sc_f = \nu_f / D_{ref}$, the base fluid thermal diffusivity is $\alpha_f = k_f / \rho_f c_f$, c_f is the base fluid heat capacity, k_f is the base fluid thermal conductivity. Other dimensionless quantities are defined $\rho_p^* = \rho_p / \rho_f$, $C_p^* = C_p / c_f$, and $C^* = C / c_f$. the subscript p denotes that of the nanoparticle material

The ‘‘incompressible’’ approximation of low Mach numbers and small relative temperature differences rendered the transport coefficients to become temperature independent if the double expansion procedure suggested in Lagerstrom [12] is carried out. Similar to the viscosity coefficient and thermal conductivity in this case, the Brownian diffusion coefficient, though it is explicitly expressed as a linear function of the temperature,

is taken as constant so that as long as the nanoparticles are indeed dilute, the dimensionless Brownian diffusion coefficient D^* is unity.

2.4 Mass diffusion equation

The volume fraction diffusion equation, which follows from the nanofluid mass conservation equation, supplemented by Fick's Law already discussed, is

$$u^* \frac{\partial \Phi}{\partial x^*} + v^* \frac{\partial \Phi}{\partial y^*} = \frac{1}{\text{ReSc}_f} \frac{\partial}{\partial y^*} \left(D^* \frac{\partial \Phi}{\partial y^*} \right) \quad (2.9)$$

The effects of thermal diffusion, which has been estimated to be small relative to mass (volume fraction) diffusion [4], is neglected at the outset, as well as its effect on energy transport in the energy equation.

2.5 The Boundary Conditions

The physical boundary conditions, in terms of the dimensionless variable, are

$$\begin{aligned} y^* = 0: u^* = 0, \theta = (T - T_\infty)/(T_0 - T_\infty) = 1, (\partial \Phi / \partial y^*) = 0 \text{ or } \Phi = \Phi_0 \\ y^* = \infty: u^* = 0, \theta = 0, \Phi = 1 \end{aligned} \quad (2.10)$$

There are two general boundary conditions in (2.10) for the volume fraction (Buongiorno, private communication 2010): In the present work the solid wall is assumed impermeable to nanoparticles. The zero flux boundary condition would yield, from the diffusion equation, a constant volume fraction throughout the boundary, $\Phi(x^*, y^*) = 1$, as solution in absence of thermal diffusion and nanoparticle sources (or sinks). In this case, the surface heat transfer rate (2.3) is accomplished by thermal conduction alone. The wall boundary condition for the temperature is the familiar one for the heat transfer problem by imposing a constant temperature for which the surface heat transfer rate is sought.

3. Thermophysical properties

Thermal conductivity and viscosity of nanofluids has been the subject of discussion and measurements by large groups of authors and laboratories (Buongiorno, et al. [13], Venerus, et al. [14]). The density and heat capacity of nanofluids, of essential interest in continuum description of nanofluid flows, on the other hand, are prevalently obtained as if the nanoparticles and base fluid were mixture of gases. Only gold nanofluid density and heat capacity are obtained systematically from molecular dynamics computations (Puliti [15], Puliti, et al. [16]). For recent review of nanofluid properties, one is referred to Paolucci & Puliti [17].

The intention here is to obtain specific forms of the properties for the two situations in which the present theory will be applied:

- (1.) water-based Alumina nanofluid for which there are entrance region experiments [2, 3] to compare with the present boundary layer theory and,
- (2.) water-based gold nanofluid for which thermophysical property results are available from molecular dynamics [15-17] and this can be compared to boundary layer results using mixture theory for the nanofluid density and heat capacity and classical theory for the nanofluid intrinsic viscosity (Einstein) and thermal conductivity (Maxwell).

3.1 Density and heat capacity

In general, nanofluid properties are expressed as a function of the volume fraction ϕ . Most, if not all, relevant heat transfer enhancement experiments using nanofluids are performed for $\phi \ll 1$. In this case, we are interested in expressing nanofluid properties in the form

$$\begin{aligned}\rho^* &= \rho / \rho_f = 1 + \phi_\infty (\rho^*)'_{\phi=0} \Phi_1 + \mathcal{O}(\phi_\infty^2) \\ \rho^* c^* &= \rho c / \rho_f c_f = 1 + \phi_\infty (\rho^* c^*)'_{\phi=0} \Phi_1 + \mathcal{O}(\phi_\infty^2)\end{aligned}\tag{3.1}$$

where $\Phi_1 = \phi_1 / \phi_\infty$ is the first order dimensionless volume fraction, the slope of the respective properties $(\rho^*)'_{\phi=0}$, $(\rho^* c^*)'_{\phi=0}$ are obtained from mixture expressions but preferably from rarely available molecular dynamics computations or measurements. In the case of mixture mimicking,

$$\begin{aligned}(\rho^*)'_{\phi=0, mix} &= \rho_p^* - 1 \\ (\rho^* c^*)'_{\phi=0, mix} &= \rho_p^* c_p^* - 1\end{aligned}\tag{3.2}$$

where $\rho_p^* = \rho_p / \rho_f$, $\rho_p^* c_p^* = \rho_p c_p / \rho_f c_f$ are the nanoparticle material density and density-heat capacity product made dimensionless by that of the base fluid. In the mixture calculation, ρ^* and $\rho^* c^*$ are identical linear functions of the volume fraction.

3.2 Transport properties

The dimensionless nanofluid viscosity coefficient and heat conductivity are also written as ascending powers of the volume fraction,

$$\begin{aligned}\mu^* &= \mu / \mu_f = 1 + \phi_\infty (\mu^*)'_{\phi=0} \Phi_1 + \mathcal{O}(\phi_\infty^2) \\ k^* &= k / k_f = 1 + \phi_\infty (k^*)'_{\phi=0} \Phi_1 + \mathcal{O}(\phi_\infty^2)\end{aligned}\tag{3.3}$$

One is referred to recent reviews of transport properties of nanofluids [13,14,17,18]. The diffusion of nanoparticle concentration is here taken to be from Brownian motion [4].

4. Perturbation analysis for small nanofluid volume concentration, $\phi_\infty \ll 1$

The thermophysical properties, expressed as function to first order in ϕ are already in a form suitable for perturbation expansion: The zeroth order is the base fluid devoid of nanoparticles. The nanofluid flow quantities are expanded as ascending powers of $\phi_\infty \ll 1$, with asterisks omitted for simplicity,

$$\begin{aligned}u &= u_0 + \phi_\infty u_1 + \mathcal{O}(\phi_\infty^2) \\ \theta &= \theta_0 + \phi_\infty \theta_1 + \mathcal{O}(\phi_\infty^2) \\ \phi &= \phi_0 + \phi_\infty \Phi_1 + \mathcal{O}(\phi_\infty^2)\end{aligned}\tag{4.1}$$

where $\phi_0 = 0$.

The zeroth order conservation equations for momentum and energy (heat transfer problem) in terms of u_0, θ_0 , respectively, are those of the Blasius and Pohlhausen problems for the base fluid [9, 10]. The first order conservation equations for u_1, θ_1 are, respectively

$$\left(u_1 \frac{\partial u_0}{\partial x} + v_1 \frac{\partial u_0}{\partial y} \right)_+ \left(u_0 \frac{\partial u_1}{\partial x} + v_0 \frac{\partial u_1}{\partial y} \right)_+ (\rho^*)'_0 \Phi_1 \left(u_0 \frac{\partial u_0}{\partial x} + v_0 \frac{\partial u_0}{\partial y} \right) = \frac{1}{\text{Re}} \left(\frac{\partial^2 u_1}{\partial y^2} + (\mu^*)'_{\phi=0} \frac{\partial}{\partial y} \left(\Phi_1 \frac{\partial u_0}{\partial y} \right) \right) \quad (4.2)$$

nanofluid inertia nanofluid viscosity

$$\left(u_0 \frac{\partial \theta_1}{\partial x} + v_0 \frac{\partial \theta_1}{\partial y} \right)_+ \left(u_1 \frac{\partial \theta_0}{\partial x} + v_1 \frac{\partial \theta_0}{\partial y} \right)_+ (\rho^* c^*)'_0 \Phi_1 \left(u_0 \frac{\partial \theta_0}{\partial x} + v_0 \frac{\partial \theta_0}{\partial y} \right) =$$

nanofluid inertia

$$\frac{1}{\text{RePr}_f} \left(\frac{\partial^2 \theta_1}{\partial y^2} + (k^*)'_{\phi=0} \frac{\partial}{\partial y} \left(\Phi_1 \frac{\partial \theta_0}{\partial y} \right) \right)_+ \frac{1}{\text{ReSc}_f} \frac{\partial}{\partial y^*} \left(\rho_p^* D^* \frac{\partial \Phi_1}{\partial y^*} c_p^* \theta_0 \right) \quad (4.3)$$

nanofluid conduction nanofluid diffusion energy transport

The nanofluid concentration equation is first order in the nanofluid volume fraction only

$$\left(u_0 \frac{\partial \Phi_1}{\partial x} + v_0 \frac{\partial \Phi_1}{\partial y} \right) = \frac{1}{\text{ReSc}_f} \frac{\partial}{\partial y} \left(D^* \frac{\partial \Phi_1}{\partial y} \right) \quad (4.4)$$

Recalling that the dimensionless diffusion coefficient is, for small temperature loading $D^* \equiv 1$.

It is noticed that nanofluid inertia effects in the first order momentum and energy equations can be recast, using the respective zeroth order Blasius and Pohlhausen momentum and energy equations, into more compact form

$$(\rho^*)'_0 \Phi_1 \left(\frac{1}{\text{Re}} \frac{\partial^2 u_0}{\partial y^2} \right), \quad (\rho^* c^*)'_0 \Phi_1 \left(\frac{1}{\text{RePr}_f} \frac{\partial^2 \theta_0}{\partial y^2} \right)$$

and moved to the respective right sides,

$$\left(u_0 \frac{\partial u_1}{\partial x} + v_0 \frac{\partial u_1}{\partial y} \right) + \left(u_1 \frac{\partial u_0}{\partial x} + v_1 \frac{\partial u_0}{\partial y} \right) = \frac{1}{\text{Re}} \frac{\partial^2 u_1}{\partial y^2} + \frac{1}{\text{Re}} \left[\Phi_1 \frac{\partial^2 u_0}{\partial y^2} ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) + (\mu^*)'_{\phi=0} \frac{\partial \Phi_1}{\partial y} \frac{\partial u_0}{\partial y} \right] \quad (4.5)$$

$$\left(u_0 \frac{\partial \theta_1}{\partial x} + v_0 \frac{\partial \theta_1}{\partial y} \right) + \left(u_1 \frac{\partial \theta_0}{\partial x} + v_1 \frac{\partial \theta_0}{\partial y} \right) = \frac{1}{\text{RePr}_f} \frac{\partial^2 \theta_1}{\partial y^2} + \frac{1}{\text{RePr}_f} \left[\Phi_1 \frac{\partial^2 \theta_0}{\partial y^2} ((k^*)'_{\phi=0} - (\rho^* c^*)'_{\phi=0}) + (k^*)'_{\phi=0} \frac{\partial \Phi_1}{\partial y} \frac{\partial \theta_0}{\partial y} \right]$$

$$+ \frac{1}{\text{ReSc}_f} \frac{\partial}{\partial y^*} \left(\rho_p^* D^* \frac{\partial \Phi_1}{\partial y^*} c_p^* \theta_0 \right) \quad (4.6)$$

It is now possible to render observations concerning the role of nanofluid in the advective-diffusive situation. In (4.6) and (4.7), inertia effects are moved to the right side so as to exhibit its competition with enhanced transport property of the nanofluid. In general, enhanced viscosity diffuses the velocity profile to render it less steep whereas, if $(\rho^*)'_{\phi=0} > 0$, inertia effects have the tendency of steepening the velocity profile. Similarly, enhanced thermal conductivity diffuses the temperature further and renders it less steep. Thermal inertia effects, characterized by the density-heat capacity product, because of the decrease of heat capacity in a nanofluid, tend also to stretch out the temperature as in the examples encountered, $(\rho^*c^*)'_{\phi=0} < 0$ and this reinforces $(k^*)'_{\phi=0}$.

The concentration diffusion equation (4.4) stands alone, with the zeroth order velocities given by the Blasius solution, subject to either of its wall boundary conditions in (2.10). It would be solved first so that Φ_1 now becomes a known function in solving (4.6) and (4.7).

5. Similar solutions

The first order solutions are expressible in similarity variables, as boundary conditions permit, in the same form as the zeroth order Blasius and Pohlhausen solutions. The correspondence between the present dimensionless variables, introduced for convenient perturbation analysis, is related to the dimensional variables found in the literature

$$\eta = \frac{y}{\sqrt{\nu_f x/U}} = \frac{y^*}{\sqrt{x^*/\text{Re}}}$$

The dimensionless stream function and velocity components are related as

$$\psi^* = \frac{\psi}{UL} = \sqrt{\frac{x^*}{\text{Re}}} f(\eta), \quad u^* = f'(\eta), \quad v^* = \frac{1}{2\sqrt{x^* \text{Re}}} (\eta f' - f)$$

These are carried over to the perturbation expansions so that these relations for the zeroth and first order velocities are similarly related to the respective stream function of the same order. The resulting ordinary equations for $f_0 = f_{\text{Blasius}}$, f_1 are, respectively

$$f_0''' + \frac{1}{2} f_0 f_0'' = 0, \quad f_0(0) = f_0'(0) = 0, \quad f_0'(\infty) = 1 \quad (5.1)$$

$$f_1''' + \frac{1}{2} (f_0 f_1'' + f_1 f_0'') = ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) \Phi_1 \frac{1}{2} f_0 f_0'' + (\mu^*)'_{\phi=0} \Phi_1' f_0'' \quad (5.2)$$

The boundary condition for the streamwise velocity is already satisfied by the Blasius solution, so that the first order problem satisfies homogeneous boundary condition, it is being driven by the inhomogeneous right side of (5.2), comprising inertia effects characterized by $(\rho^*)'_{\phi=0}$ and the velocity-profile stretching or smoothing effects of enhanced viscosity characterized by $(\mu^*)'_{\phi=0}$. These two mechanisms are competing since they are of opposite sign.

The zeroth and first order energy equations become, after the similarity variable transformation,

$$\theta_0'' + \frac{\text{Pr}_f}{2} f_0 \theta_0' = 0, \quad \theta_0 = 1, \quad \theta_0(\infty) = 0 \quad (5.3)$$

$$\theta_1'' + \frac{\text{Pr}_f}{2} (f_0 \theta_1' + f_1 \theta_0') = - \left[((k^*)'_{\phi=0} - (\rho^* c^*)'_{\phi=0}) \Phi_1 \theta_0'' + (k^*)'_{\phi=0} \Phi_1' \theta_0' \right] - \frac{\rho_p^* c_p^*}{\text{Sc}_f} D^* (\Phi_1' \theta_0)' \quad (5.4)$$

The inhomogeneous equation for θ_1 satisfies homogeneous boundary conditions as the temperature boundary conditions of the problem are satisfied by θ_0 .

The nanofluid properties are dependent on the nanoparticle volume concentration, thus it is necessary to consider the volume concentration diffusion equation.

5.1 The case of a solid wall with zero nanoparticle flux

The volume diffusion problem in this case is

$$\Phi_1'' + \frac{\text{Sc}_f}{2} f_0 \Phi_1' = 0, \quad \Phi_1'(0) = 0, \quad \Phi_1(\infty) = 1 \quad (5.5)$$

In the absence of sources (or sinks) in the interior of the nanofluid and on the boundaries, (5.5) yields an exact solution

$$\Phi_1(\eta; \text{Sc}_f) = 1 \quad (5.6)$$

(much like the Crocco Integral for the fluid energy equation in the case of an insulated wall and $\text{Pr} = 1$, [9]). In this special case, the problem is reduced to constant thermophysical properties dependent only on the constant volume fraction ϕ_∞ . As such, it loses the ability of regulating heat transfer at the wall via thermal energy transport through concentration diffusion according to (2.7). Thus the surface heat transfer can be ascertained from boundary layer theory using nanofluid thermophysical properties at the prevailing ϕ_∞ . This has been found to be the case in certain measurements according to Prabhat, et al. [19,20] although no information on the nanoparticle volume fraction distribution is available

5.2 The momentum and thermal problems for zero nanoparticle volume flux at solid wall

The boundary condition of zero flux at the wall produces $\Phi_1(\eta) = 1$ throughout the entire boundary layer so that $\Phi_1'(\eta) = 0$, thus (5.2) and (5.4) simplify to, respectively,

$$\begin{aligned}
f_1''' + \frac{1}{2}(f_0 f_1'' + f_1 f_0'') &= ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) \frac{1}{2} f_0 f_0'' \\
f_1(0) = f_1'(0) = 0, \quad f_1'(\infty) &= 0
\end{aligned} \tag{5.9}$$

and

$$(5.10)$$

where use is made of the zeroth order momentum (5.1) and energy (5.3) equations to arrive at the “advective” form on the right sides on (5.9) and (5.10), respectively.

It is noticed that if we define

$$F_1 = f_1 / ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) \tag{5.11}$$

and substitute into (5.9) to obtain

$$\begin{aligned}
F_1''' + \frac{1}{2}(f_0 F_1'' + F_1 f_0'') &= \frac{1}{2} f_0 f_0'' \\
F_1(0) = F_1'(0) = 0, \quad F_1'(\infty) &= 0
\end{aligned} \tag{5.12}$$

then (5.12) can be solved for F_1 as a “universal function” independent of thermophysical properties. Less fortunate in this respect is (5.10) which requires specification of thermophysical properties prior to its solution.

5.3 Shear stress at the wall

Denoting the wall condition by the subscript 0, the Newtonian shear stress at the wall is

$$\tau_0 = \left(\mu \frac{\partial u}{\partial y} \right)_0 = \mu_f \mu_0^* u_\infty \sqrt{u_\infty / \nu_f} \chi f''(0)$$

the representation of the nanofluid viscosity coefficient is discussed in Section 3. Inserting the perturbation representations, to first order in ϕ_∞ , the ratio of nanofluid shear stress to that of base fluid is

$$\tau_0 / \tau_{0,f} = 1 + \phi_\infty \left[(\mu^*)'_{\phi=0} + f_1'(0) / f_0''(0) \right]$$

where base fluid shear stress is $\tau_{0,f} = \mu_f u_\infty \sqrt{u_\infty / \nu_f} X f_0''(0)$. In the similar solution for both water and for nanofluid flow, the shear stress decay as $X^{-1/2}$. The shear stress ratio or enhancement is independent of X although experimentally this may not be the case. The local skin friction coefficient is defined as the local shear stress normalized by the free stream dynamic pressure. The respective base fluid and nanofluid skin friction coefficients are

$$C_{f_f} = \tau_{0,f} / (1/2) \rho_{\infty,f} u_\infty^2, \quad C_f = \tau_0 / (1/2) \rho_{\infty,f} [1 + \phi_\infty (\rho^*)'_{\phi=0}] u_\infty^2$$

With the appropriate definition of the skin friction coefficient, which necessarily involves the free stream density of the nanofluid, it is thus more physically straightforward to discuss the skin friction enhancement directly in terms of their ratios rather than the ratio of skin friction coefficients.

5.4 Surface heat transfer rate

The surface heat transfer rate at the wall from (2.2) includes the heat transfer owing to diffusion currents. In the present case of zero flux of nanoparticles across a solid wall for which the volume concentration is uniform thus heat transfer at the wall is accomplished by heat conduction alone. Using the representation of nanofluid thermal conductivity of Section 3 and the perturbation representations, we obtain the heat transfer rate enhancement ratio, to first order in ϕ_∞

$$q_0 / q_{0,f} = 1 + \phi_\infty [(k^*)_0' + \theta_1'(0) / \theta_0'(0)]$$

where the base fluid heat transfer rate is

$$q_{0,f} = k_f (T_0 - T_\infty) \theta_0'(0) \sqrt{u_\infty / \nu_f} X$$

From the definition of the local heat transfer coefficient

$$h_x = q_0 / (T_0 - T_\infty)$$

the ratio of the surface heat transfer rates is the same as that of heat transfer coefficients

$$h_x / h_{x,f} = q_0 / q_{0,f}$$

provided that the wall temperatures of the nanofluid and of the base fluid are the same.

5.5 Brief description of numerical procedure

Although the Blasius and Pohlhausen problems are well known, they are nevertheless solved to provide numerical consistency for the first order problems. Both (5.10) and (5.11) are inhomogeneous differential equations with homogeneous boundary conditions. They are numerically solved using Octave, similar to Matlab, by writing a single function to describe the entire system $f_0, F_1, \theta_0, \theta_1$. The boundary value problem is

then solved as an initial value problem by the shooting method. As there are four unknown conditions at $\eta = 0$ which are required to solve the entire system $(f_0'', F_0'', \theta_0', \theta_1')$, each of the four conditions is solved individually, one at a time. Initially, the Blasius problem is considered and forward integration is carried out in a loop as the guess for f_0'' is updated. Once the stand-alone f_0'' solved, then θ_0 or F_1 may be solved in the same manner, θ_1 is solved last as the three functions are needed as variable coefficients and inhomogeneous input. The relevant code is documented but not reproduced here.

5.6 Application to specific nanofluids

For alumina $(\rho^*)'_{\phi=0,mix} = 2.89$, $(\rho^*c^*)'_{\phi=0,mix} = -0.18$. For gold-water nanofluid $(\rho^*)'_{\phi=0,mix} = 18.3$, $(\rho^*c^*)'_{\phi=0,mix} = -0.42$. Interpolating results of Puliti, et al. [16] for heat capacity in J/mol,K to kJ/g,K via effective nanofluid molecular weight vs. effective nanoparticle volume fraction, it is obtained $(\rho^*)'_{\phi=0,MD} = 18.7$, $(\rho^*c^*)'_{\phi=0,MD} \cong -2.37$. In the case of dilute, spherical nanoparticles, the nanofluid behaves like a Newtonian fluid, Einstein's result $(\mu^*)'_{\phi=0,Einstein} = 2.5$ is satisfactory. The classical result of Maxwell gives $(k^*)'_{\phi=0,Maxwell} = 3$, but Wen & Ding's (2004) measurements give $(k^*)'_{\phi=0,W\&D} \cong 6$ for water based Alumina.

Molecular dynamics simulation for viscosity and thermal conductivity of gold nanoparticles appear only in the thesis of Puliti (2012) and are subject to interpretation to bring the results into practical form. The results suggest that $0 \leq (\mu^*)'_{\phi=0,MD} \leq 10$, based on points for $\phi = 0$ and $\phi = 0.10$, including the full expanse of error bars (Puliti 2012, Figure 3.30 and Table 3,3). There are, however some doubts engendered by the incorrect value produced by the simulation for pure water at $\phi = 0$ and the negative slope of the computed value of $(\mu^*)'_{\phi=0,MD}$ at $\phi = 0.01$. The molecular dynamics result for gold nanoparticle thermal conductivity [15] suggests that $(k^*)'_{\phi=0,MD} = 4.3$ between $\phi = 0$ and $\phi = 0.10$; but for small concentrations to $\phi = 0.01$, $(k^*)'_{\phi=0,MD} \leq 20$. The summary of properties values in applications are given in the following table

Case I. Alumina,mix	Case II. Gold-Water,mix	Case III. Gold-Water,MD
$(\rho^*)'_{\phi=0,mix} = 2.89$	$(\rho^*)'_{\phi=0,mix} = 18.3$	$(\rho^*)'_{\phi=0,MD} = 18.7$
$(\rho^*c^*)'_{\phi=0,mix} = -0.18$	$(\rho^*c^*)'_{\phi=0,mix} = -0.42$	$(\rho^*c^*)'_{\phi=0,MD} \cong -2.37$
$(\mu^*)'_{\phi=0,Einstein} = 2.5$	$(\mu^*)'_{\phi=0,Einstein} = 2.5$	$(\mu^*)'_{\phi=0,MD} = 10$
$(k^*)'_{\phi=0,W\&D} \cong 6$	$(k^*)'_{\phi=0,Maxwell} = 3$	$(k^*)'_{\phi=0,MD} = 20$

The thermophysical properties representing the competition between conductive, diffusive effects and advective effects of inertia are reproduced below

	Case I. Alumina,mix	Case II. Gold-Water,mix	Case III. Gold-Water,MD
$(\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}$:	- 0.39	- 15.80	- 8.70

$$(k^*)'_{\phi=0} - (\rho^*c^*)'_{\phi=0}: \quad +6.18 \quad +3.42 \quad +22.37$$

6. The normalized universal velocity function

The universal function $F_1(\eta)$ from (5.12) is the first order velocity function, normalized by material properties as indicated, is shown in Figure 1 together with the zeroth order Blasius velocity function f'_0 of (5.1).

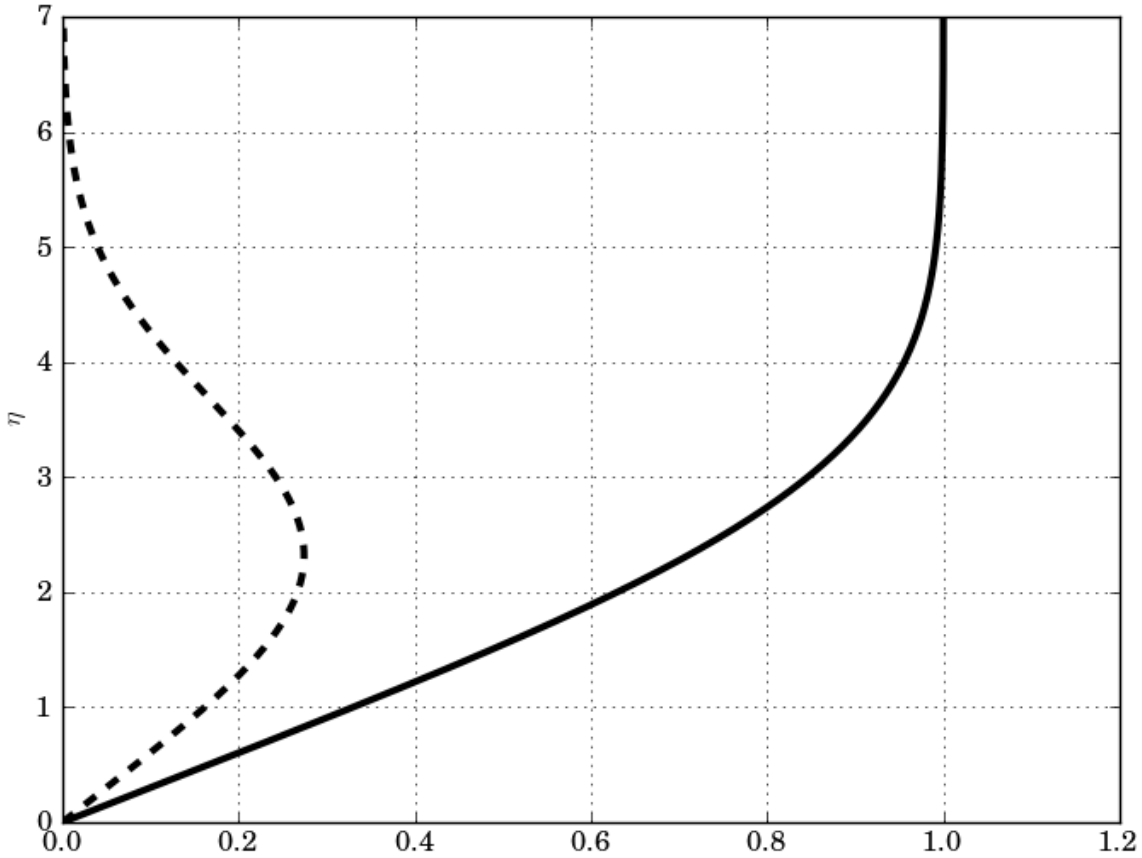


Figure 1. The first order universal velocity function F_1' -----, the zeroth order Blasius velocity function f_0' ——— vs. the Blasius similarity variable η , where $F_1 = f_1 / ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0})$.

The first order velocity profile decreases the steepness of the Blasius velocity profile as increasing function of the volume fraction as the stretching factors are negative for all three cases considered. The enhanced viscosity effect is to stretch the velocity profile rendering it less steep, whereas the inertia effect is to steepen the velocity profile. The imbalance of these two effects, according to the material properties obtained, is to decrease the steepness of the overall velocity profile.

7. Specific cases of application: velocity and temperature profiles

7.1 Case I: alumina-water nanofluid

The nanofluid effects on the velocity profiles is best brought out by the difference between the nanofluid velocity and that of the base fluid Blasius profile, in terms of the universal velocity function,

$$\begin{aligned} [f'(\eta) - f_0'(\eta)] / \phi_\infty &= f_1'(\eta) = ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) F_1(\eta) \\ ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) &= -0.39 \end{aligned}$$

The modification of nanofluid velocity profile over that of the base fluid conceivably could be measured in laboratory experiments. In Figure 2 is shown the velocity profile difference for Case I for small volume fraction

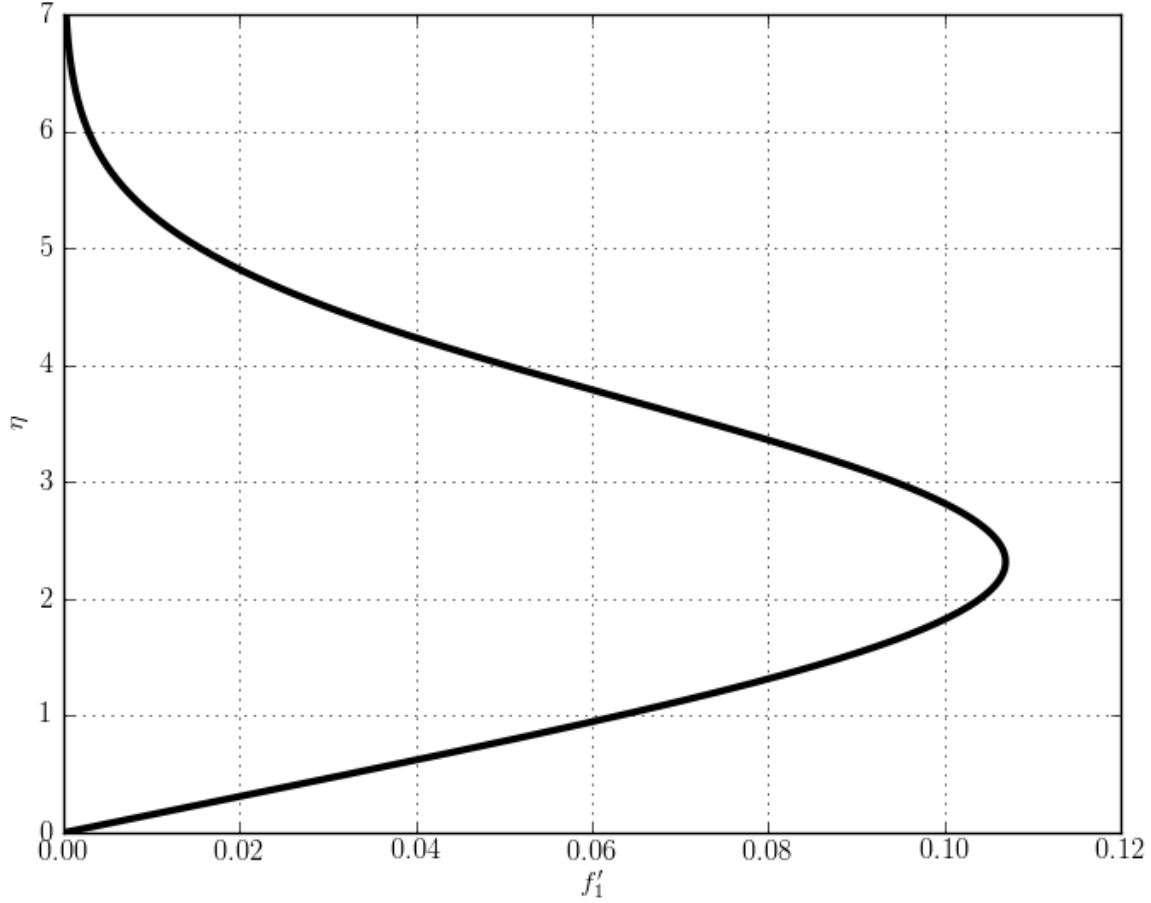


Figure 2. The first order velocity profile $f'_1(\eta)$ for alumina-water nanofluid as function of the similarity variable η , Case I

The maximum value of $f'_1(\eta) = \mathcal{O}(0.1)$ so that for small volume fraction the over all profile would be graphically difficult to visualize, although the profile $f'_1(\eta)$ itself is numerically discernable.

The temperature profiles from (5.10) need the input of property values in their solution as well as the velocity profiles. The case for alumina is shown in Figure 3 (Case I). In this case the overall temperature profile

$$\theta(\eta) = \theta_0(\eta) + \phi_\infty \theta_1(\eta)$$

is rendered less steep with increasing volume fraction. In the much simpler case of Rayleigh-Stokes approximation [1], it was pointed out, as with the velocity profile, there is the competition between the profile stretching effect of enhanced transport property (thermal conductivity in this case) and the temperature profile steepening effect of inertia through the density-heat capacity product. It is more intricate in the boundary layer problem here than in the thermal Rayleigh-Stokes problem [1] as the velocity profiles are also involved, as (5.10) clearly shows. Again, the nanofluid temperature profile effect is best shown as the difference between nanofluid temperature and that of the base fluid which correspond to the Pohlhausen profile [10]

$$[\theta(\eta) - \theta_0(\eta)] / \phi_\infty = \theta_1(\eta),$$

and this, again, could be measured in the laboratory. The maximum value of the first order temperature profile is nearly 3.0 (Figure 3).

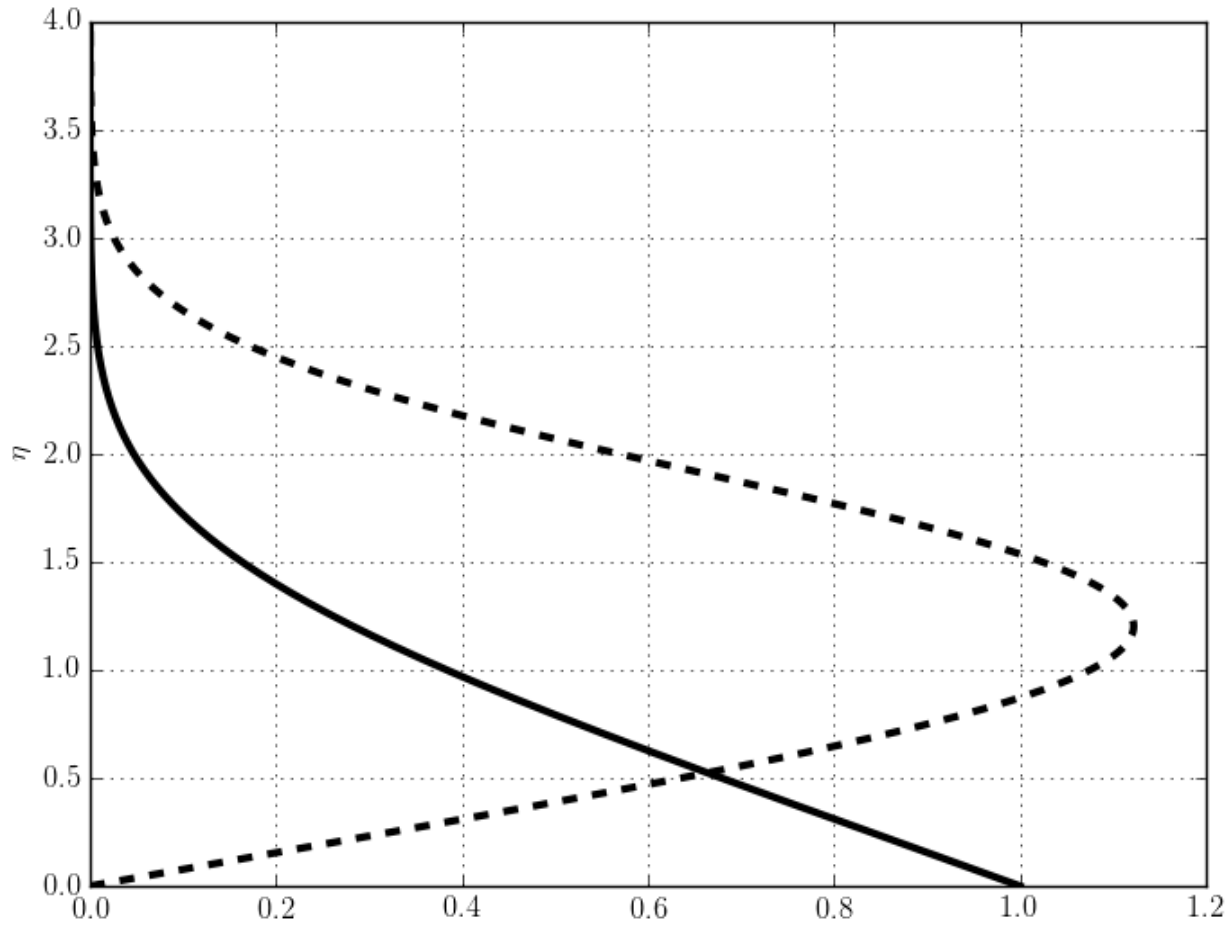


Figure 3. The zeroth order Pohlhausen profile θ_0 ———, first order temperature profile θ_1 ----- as function of the similarity variable, η Alumina-water (Case I), $Pr_f = 7$

7.2 Case II: Gold-water nanofluid (mix)

The first order perturbation velocity profile for Case II is

$$[f'(\eta) - f'_0(\eta)]/\phi_\infty = f'_1(\eta) = ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) F_1(\eta),$$

where $(\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0} = -15.80$, is shown in Figure 4. It has a maximum value greater than 4.0 and is significantly greater than that for Case I. It gives rise to a steepening of the overall velocity profile.

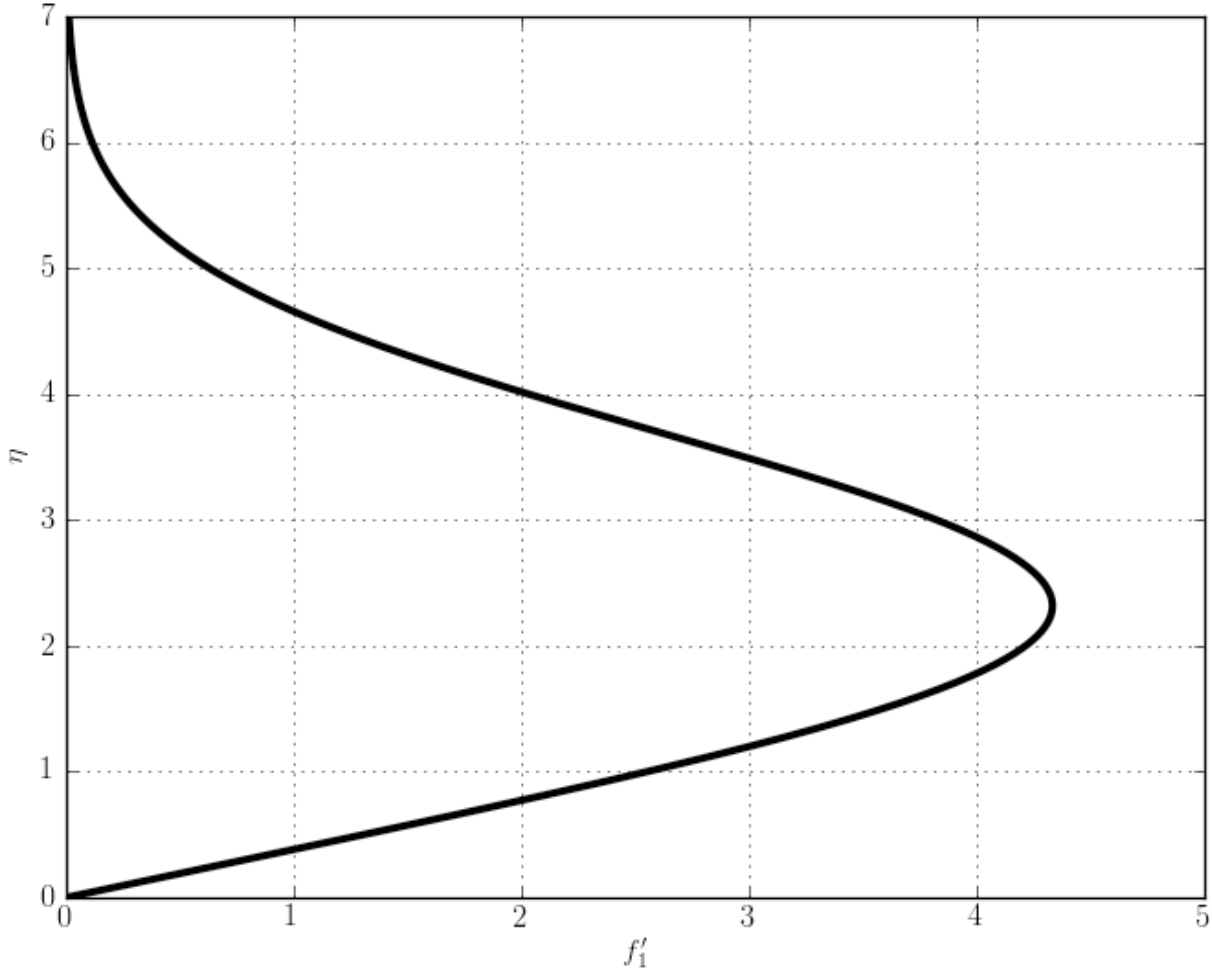


Figure 4. First order velocity profile $f'_1(\eta) = ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) F_1(\eta)$, as function of the similarity variable η , Case II

The first order temperature profile $\theta_1(\eta)$ for Case II is shown in Figure 5, along with the zeroth order Pohlhausen profile. In this case, the nanofluid effect is to steepen the dimensionless temperature gradient at the wall. The maximum magnitude of $\theta_1(\eta)$ is about -2.0 and again, $[\theta(\eta) - \theta_0(\eta)] / \phi_\infty = \theta_1(\eta)$ appears to be a measurable quantity.

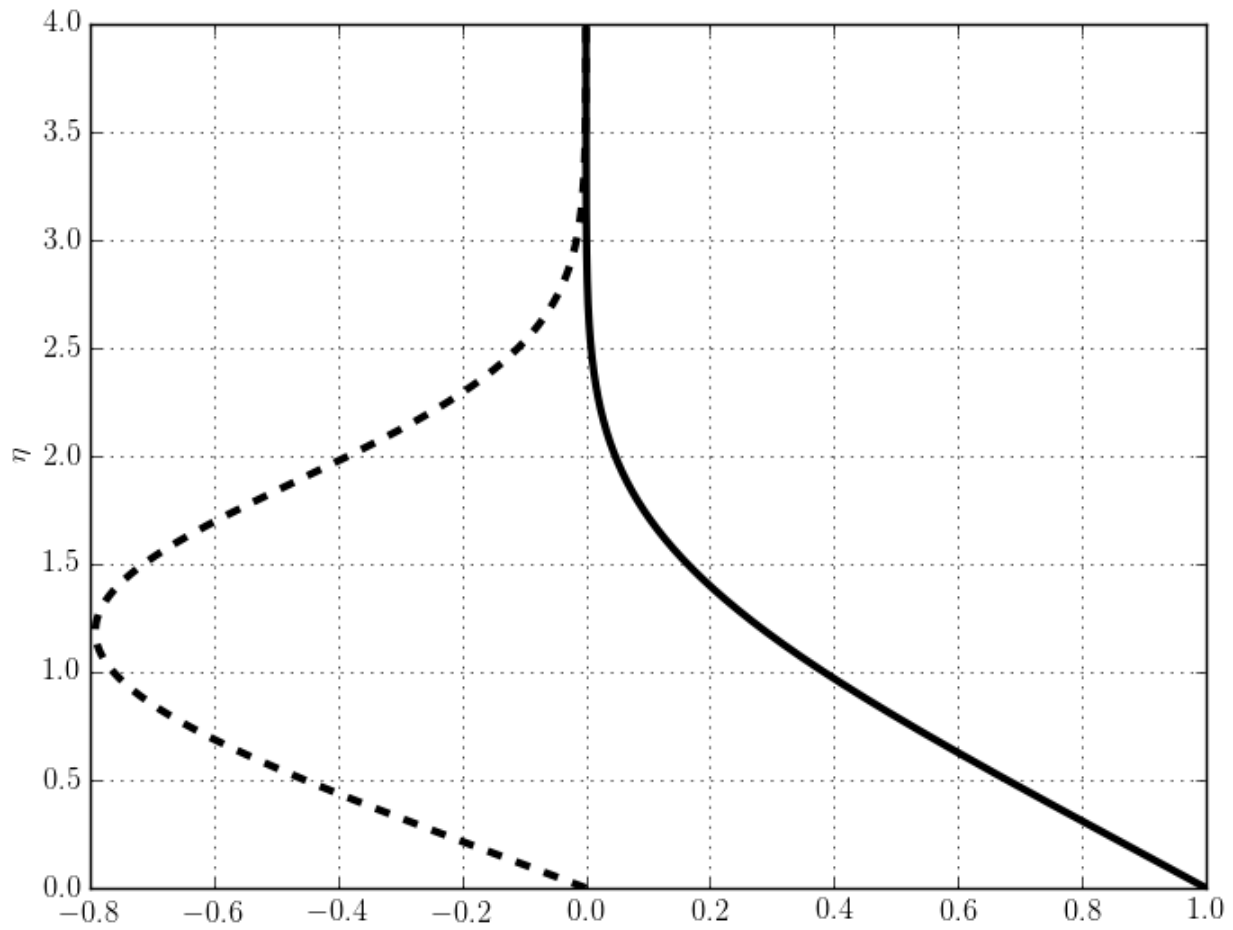


Figure 5. Gold-water nanofluid temperature profiles vs. similarity variable η , Case II. First order profile $\theta_1(\eta)$ ----- is negative and is shown on the left, the zeroth order Pohlhausen profile is on the right θ_0 —————. $Pr_f = 7$

7.3 Gold-water nanofluid (MD) Case III

The velocity difference profile for Case III,

$$[f'(\eta) - f'_0(\eta)]/\phi_\infty = f'_1(\eta) = ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) F_1(\eta)$$

$$((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) = -8.70$$

is shown in Figure 6, with a maximum value of about 2.40. It again, contributes to the steepening of the overall velocity profile.

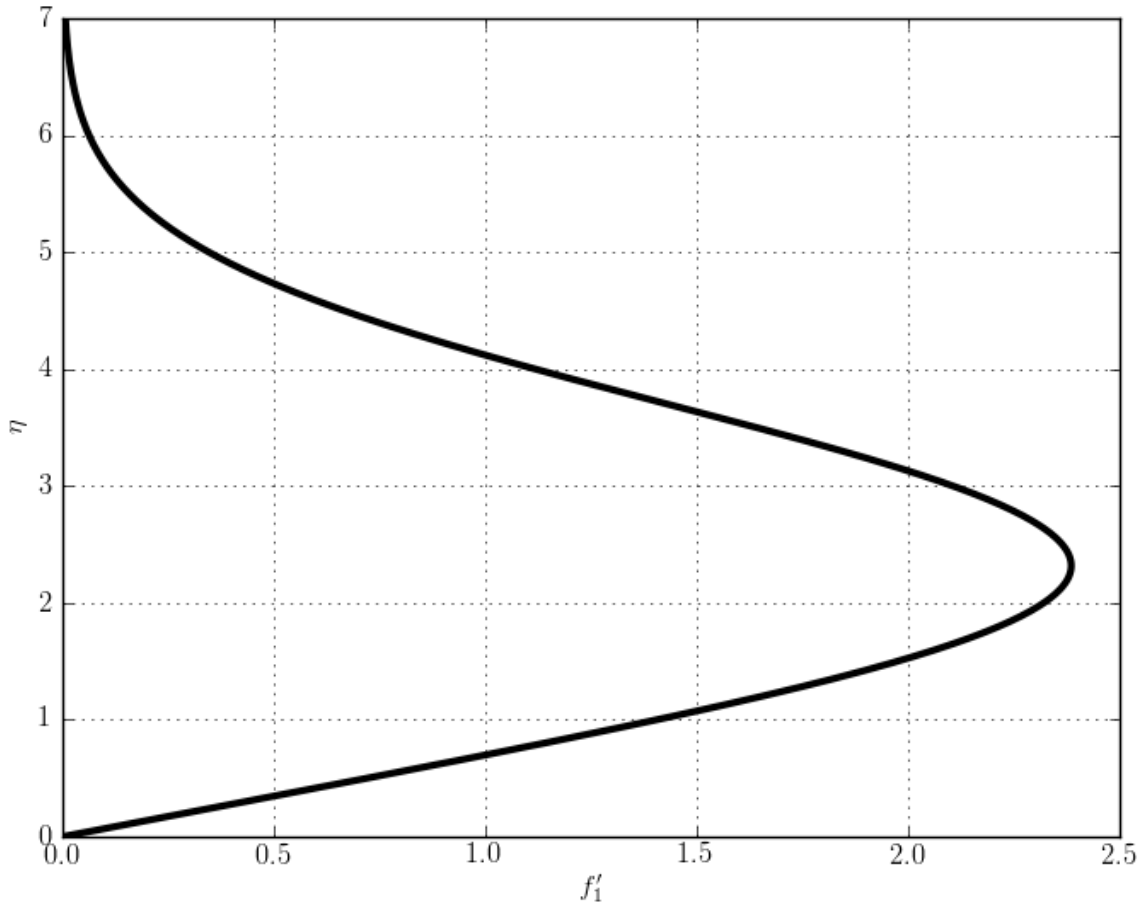


Figure 6. The first order velocity profile $f'_1(\eta) = ((\mu^*)'_{\phi=0} - (\rho^*)'_{\phi=0}) F_1(\eta)$ for Gold-water nanofluid. Case III.

The first order temperature profile for Case III,

$$[\theta(\eta) - \theta_0(\eta)]/\phi_\infty = \theta_1(\eta)$$

is shown in Figure 7, with a maximum value nearly 9.0 The nanofluid effect of the temperature profile is to decrease the gradient at the wall. Although not obvious in the boundary layer problem, the Rayleigh-Stokes problem [1] tells us that it is attributable to the large thermal conductivity effect of smoothing out the temperature profile.

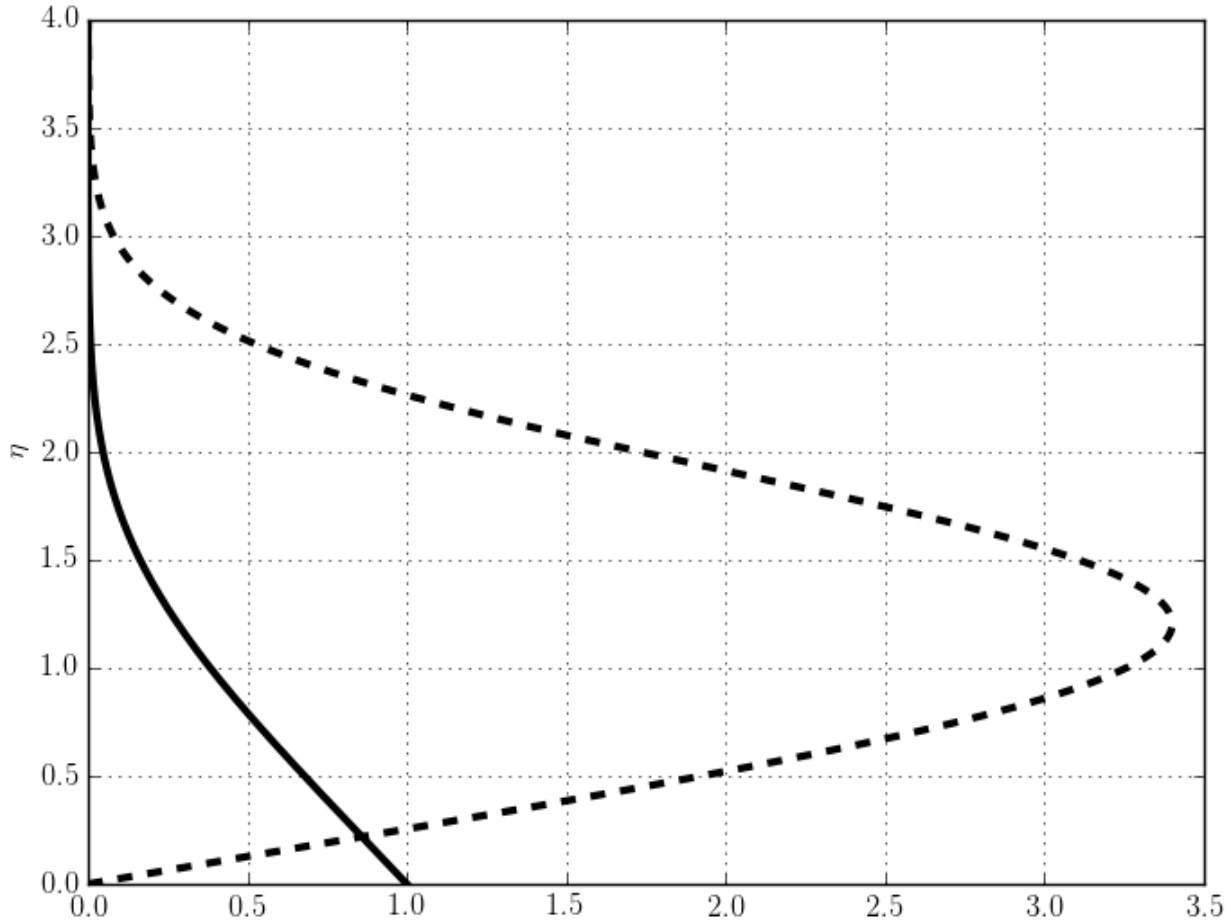


Figure 7. Gold-water (MD) nanofluid temperature profiles vs. similarity variable η , Case III. First order profile $\theta_1(\eta)$ ----- is positive, the zeroth order Pohlhausen profile θ_0 —————. $Pr_f = 7$

8. Surface heat transfer rate and skin friction

In the previous section the nanofluid effects on the velocity and temperature profiles are illustrated, owing to the next effect of steepening due to inertia and profile stretching effects of enhanced transport properties. The impact of transport effects on the resulting velocity and temperature profiles are discussed in this section and has an explicit effect on the enhancement of nanofluid skin friction rise and heat transfer relative to the base

fluid, overcoming the profile-stretching effects of enhanced nanofluid transport properties in the respective conservation equations of momentum and static enthalpy.

The general relations for skin friction and surface heat transfer rate are rewritten in terms of relations normalized by the corresponding values of the base fluid. The relative skin friction enhancement is, to first order in ϕ_∞

$$\tau^* - 1 = \phi_\infty \left[(\mu^*)'_{\phi=0} + f_1''(0)/f_0''(0) \right] = \phi_\infty (\tau^*)'_{\phi=0}$$

where $\tau^* = \tau_0 / \tau_{0,f}$ and the base fluid shear stress is $\tau_{0,f} = \mu_f u_\infty \sqrt{u_\infty / \nu_f} \chi f_0''(0)$. The slope $(\tau^*)'_{\phi=0}$ requires the input of transport property and the solution of the boundary layer similarity solutions up to the first order.

The heat transfer rate at the wall for zero flux of nanoparticles across the wall is accomplished by heat conduction alone. Using the representation of nanofluid thermal conductivity and the perturbation representations, we obtain the enhanced heat transfer rate relative to the base fluid, to first order in ϕ_∞

$$q^* - 1 = \phi_\infty \left[(k^*)'_0 + \theta_1'(0)/\theta_0'(0) \right] = \phi_\infty (q^*)'_{\phi=0}$$

where the surface heat transfer rate is normalized by that of the base fluid

$$q^* = q_0 / q_{0,f}, \quad q_{0,f} = k_f (T_0 - T_\infty) \theta_0'(0) \sqrt{u_\infty / \nu_f} \chi$$

The results in the rise in shear stress and heat transfer enhancement are presented in the following table, for the three cases discussed.

	Case I: alumina(mix)	Case II: gold/water(mix)	Case III: gold/water (MD)
$(q^*)'_{\phi=0}$	3.98	4.44	13.90
$(\tau^*)'_{\phi=0}$	2.70	10.40	14.35

8.1 Case I

The comparisons with experiments in alumina-water nanofluids (Case I) are made for the highest Reynolds numbers. This ensures that the measurements are in the entrance region of channels [3] and tubes [4], which corresponds to the present boundary layer region rather than in the developed region. The comparisons are shown here in terms of q^* in Figures 8 and 9, respectively. The theory from use of mixture properties underestimates measurements.

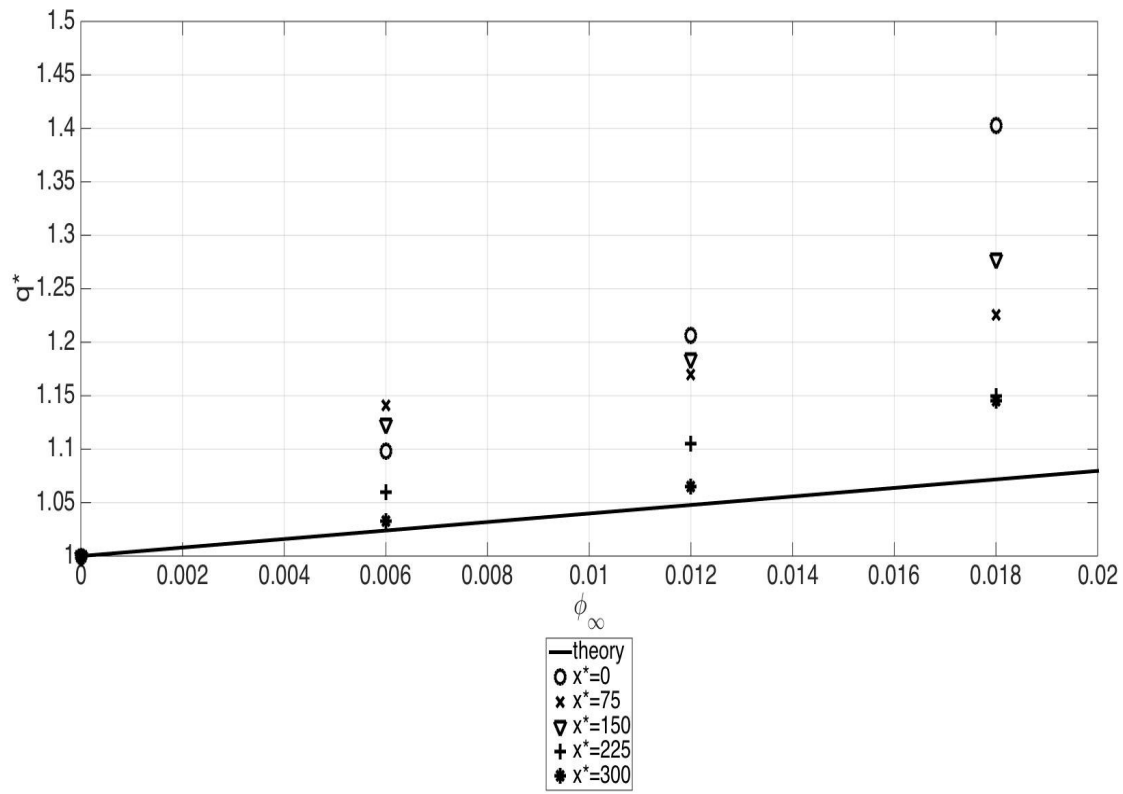


Figure 8. Comparison of theory (mix) for alumina-water nanofluids, measurements are those of Jung, et al. [3] at Reynolds number of 287 (characteristic length based on channel height).

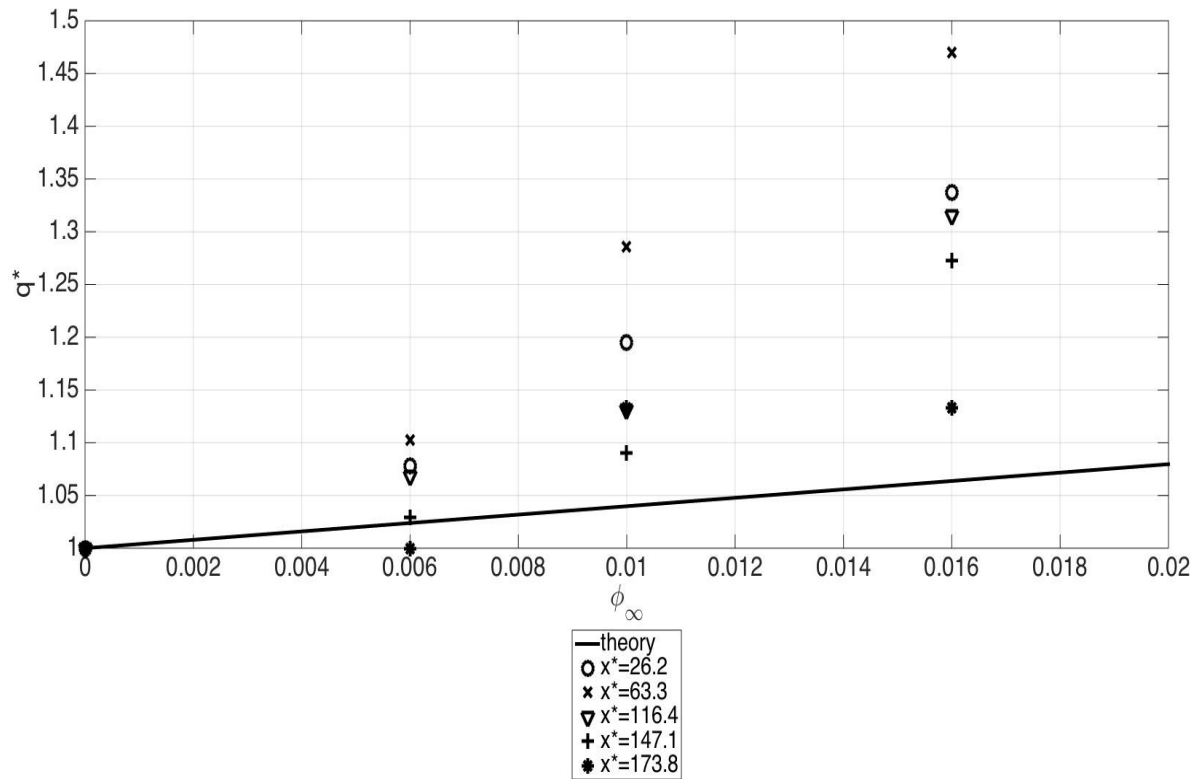


Figure 9. Comparison of theory (mix) for alumina-water nanofluids, measurements are those of Wen & Ding [2] at Reynolds number of 1600 (characteristic length based on tube diameter).

The underestimation by the theory of experimental enhancement in the entrance region can be attributed to several factors. One is believed to be the use of mixture and “conventional” thermophysical properties which consistently underestimate the heat transfer enhancement from that of experiments. (MD calculation of thermophysical properties for alumina-water is not available). The other reasons are that, the upstream detailed initial conditions of nanofluid velocity and temperature in the experiments are not known. The entrance of channels and tubes are fed by tubes which most certainly has developed velocity and temperature profiles which advects flow and temperature in the experimental channels or tubes. The measurements all indicate a high degree of non-similarity in the streamwise development from the entrance leading edge. Experimental results contrast with theory, which assumes uniform entrance region velocity and temperature which allowed the simplicity of similar solutions. In this case, the theory gives a minimum level of enhancement and forms the basis for further extensions into realistic thermophysical properties, when available, and more realistic upstream conditions based upon experimentation.

The relative heat transfer enhancement and skin friction rise for Case I is shown in Figure 10.

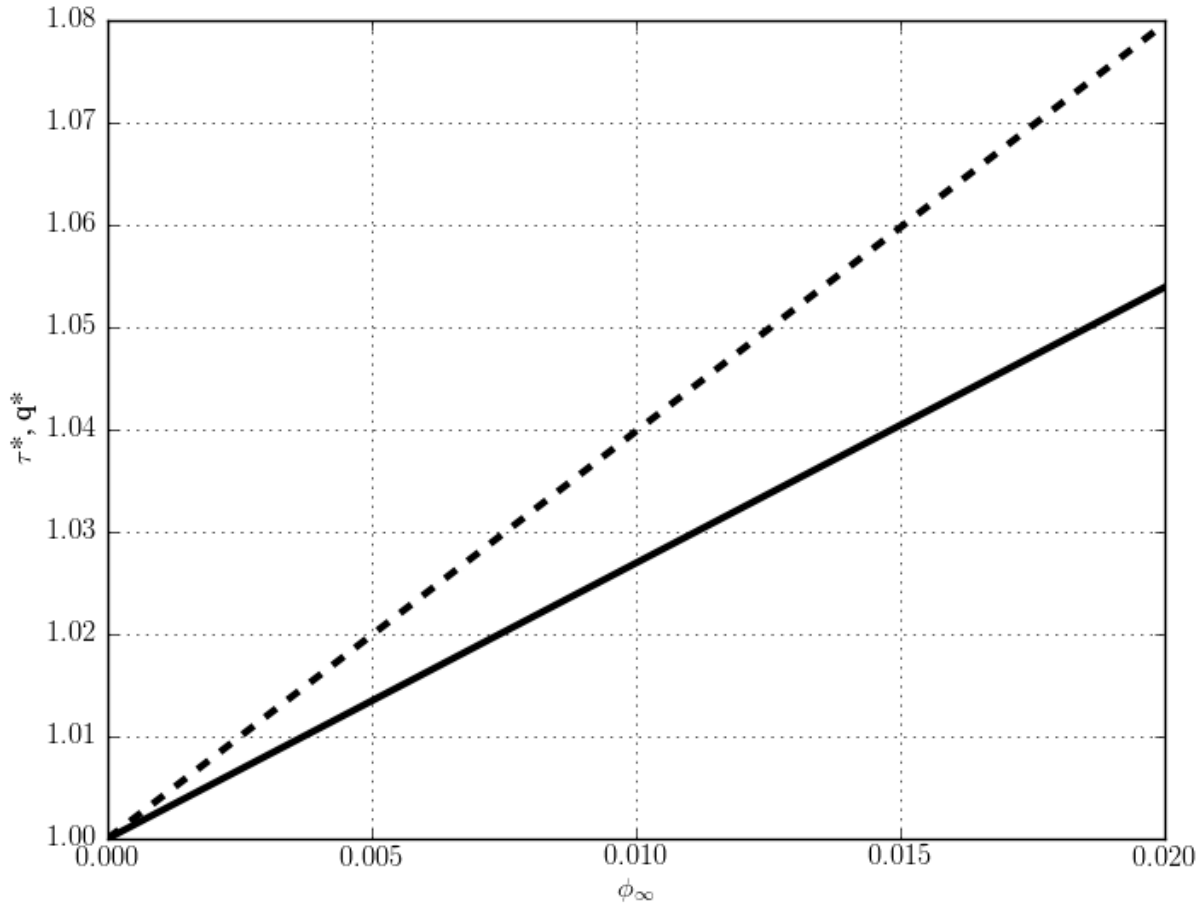


Figure 10. Heat transfer enhancement and skin friction rise vs. volume fraction. Case I, alumina-water(mix) nanofluid.

q^* -----, τ^* _____

The Case I result, according to the boundary layer solution and the use of thermophysical properties stated, actually give rise to a relative larger heat transfer enhancement than the rise in skin friction, a scenario more optimistic than that described in Venerus, et al. [14]. This is because of the leading edge effects in the present theory, which accounts for the inertia effects, the nanofluid density in momentum convection and the nanofluid density-heat capacity in the convection of static enthalpy. If one were to consider fully developed flow, where no such convection effects are present, would give a pessimistic account of nanofluids in heat transfer enhancement. In fact, Ding, et al. [21] proposed heat exchanger using an assembly of entrance region heat transfer tubes or channels. The present analysis and of Liu [1], which brought out the completion between inertia effects and enhanced transport effect in the stretching or squashing of the temperature profile and velocity profiles, which in turn, augment the explicit transport effect in the surface heat transfer rate and skin friction. This emphasize that transport properties play a dual, opposing role when convection effects are present.

8.2 Gold nanofluid (mix), Case II

Although there are no experiments performed with gold nanoparticles, there exist (MD) calculations [15-17] of properties. The results of their use in boundary layer analysis can be compared to that from mixture calculation (mix) of properties and conventional transport properties. The profile results, Figures 4 and 5, which reflects the competition between nanofluid convection and diffusive effects on the profiles, together the surface heat transfer rate and skin friction rise, reflecting the enhance nanofluid transport properties, as in Figure 10 for Case I, are given in Figure 11 for Case II

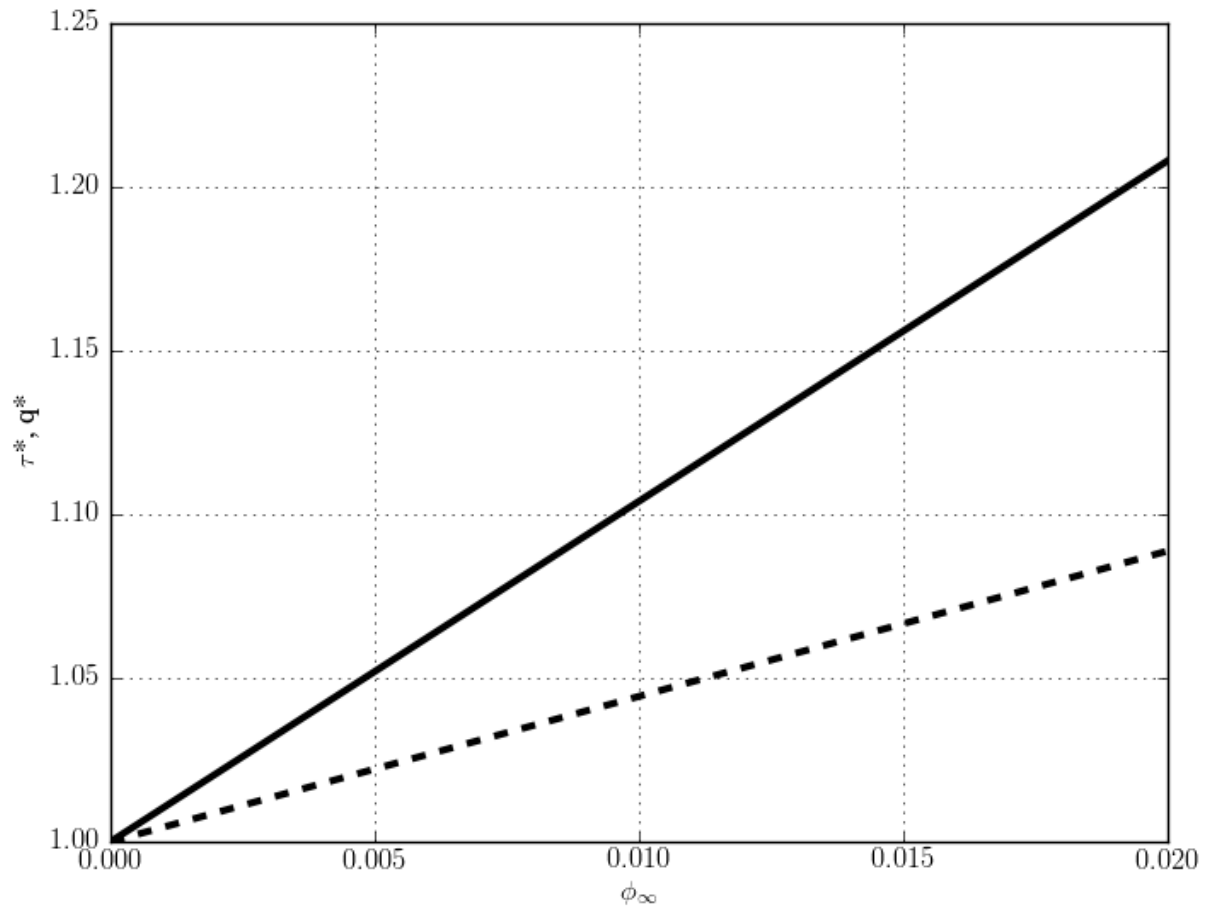


Figure 11. Heat transfer enhancement and skin friction rise vs. volume fraction. Case II, gold-water(mix) nanofluid.

q^* -----, τ^* ———

8.3 Gold nanofluid, Case III

The corresponding surface heat transfer rate and skin friction rise for Case III is shown in Figure 12.

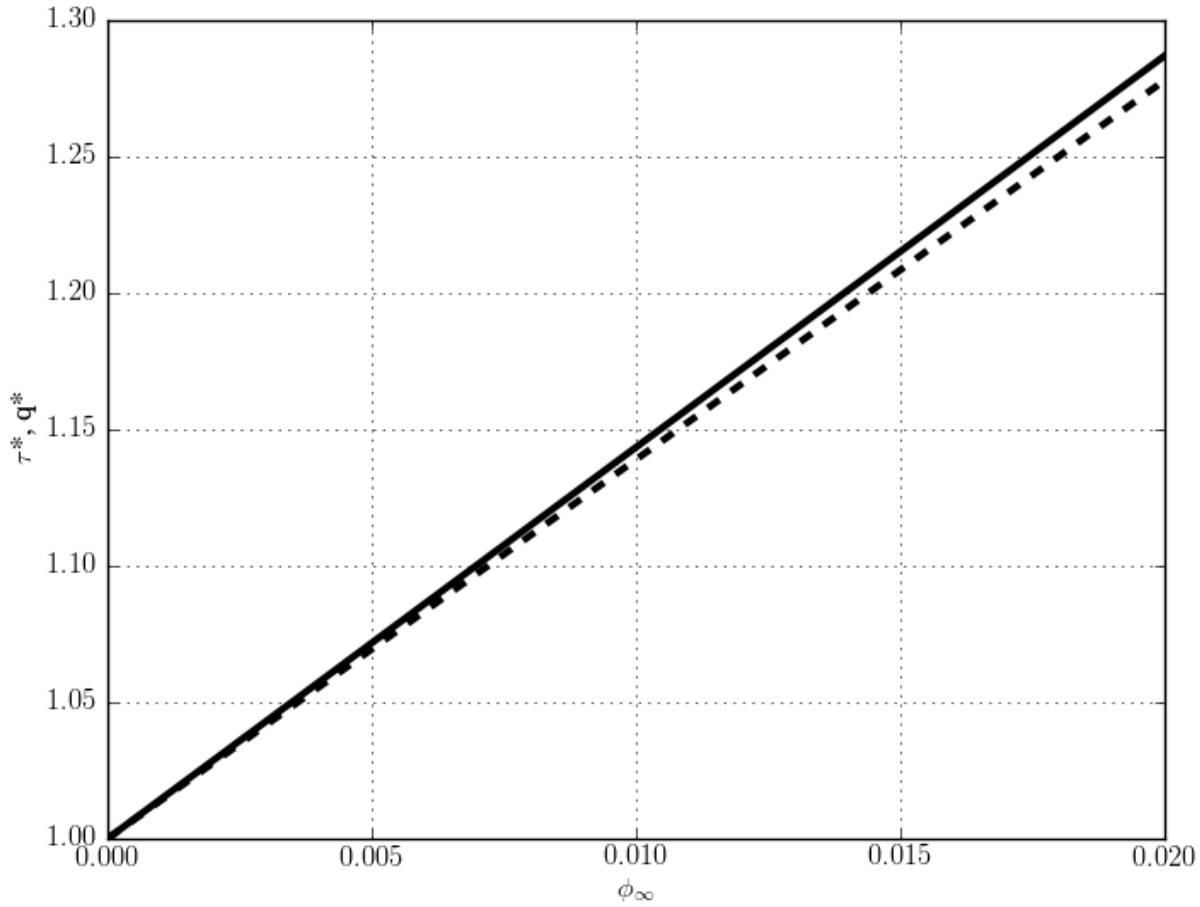


Figure 12. . Heat transfer enhancement and skin friction rise vs. volume fraction. Case III, gold-water(MD) nanofluid.

τ^* ———, q^* -----

The results from Figures 11 and 12, Cases II and III, respectively, indicate that mixture approach and conventional use of Einstein and Maxwell viscosity and thermal conductivity relations considerably underestimates both the skin friction rise and enhanced heat transfer compared to molecular dynamics results for thermophysical properties. Use of molecular dynamics obtained [15-17] thermophysical properties in the present boundary layer theory appreciably enhanced both the surface heat transfer rates and skin friction. The relative enhancements are very nearly one-to-one (Figure 12) and thus removes the pessimism expressed in Venerus, et al. [14] concerning nanofluids, at least for gold-water nanofluids. Also, their [14] conclusion is based largely on fully developed flow which does not have the benefit of the developing leading effect of bring into play convective inertia effects [1-3, 21] which is brought out by the present work.

8.3 Remarks on gold nanofluid experiments

An experimental study of convective heat transfer of gold-water nanofluid in a tube of diameter 2.27mm and 580mm length is reported by Sabir, et al. [22]. As in experiments [2,3] designed for heat transfer studies, the entrance velocity and temperature profiles are not well defined. The reported [22] enhanced surface heat transfer rate , in terms of the present notation, attained values of 1.08, 1.15 and 1.23 for nanofluid volume fraction of 0.00015, 0.00045 and 0.000667, respectively. In this case, data points would almost appear along the vertical axis in Figure 12 for these unusually minute volume fractions. The thermal conductivity is not explicitly presented in these specific experiments [22]. Thus it is difficult to uncover whether the thermal conductivity

enhancement mechanism falls within and aligned with the extensive measurements discussed in Buongiorno, et al. [13].

9. Concluding remarks

It is found that the use of molecular dynamics properties for gold-water nanofluids considerably increased the heat transfer enhancement relative to results using “conventional” properties. Furthermore, heat transfer enhancement is comparable to the enhanced skin friction rise, whereas using conventionally obtained properties the enhanced skin friction rise is ominously larger than the heat transfer enhanced for gold nanoparticles. To fully appreciate the potential in the use of nanofluids in heat transfer enhancement, further molecular dynamics computations of properties of nanofluids, including transport properties, accompanied by careful laboratory experiments on velocity and temperature profiles are thus suggested. Possible theoretical extensions would be to include upstream shear flows prior to the leading edge problem, as would be found in experimental heat transfer measurements where the micro channels and tubes are fed by upstream nanofluid in tubes.

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